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Coordination across scales: A multifractal perspective on scientific modeling of movement

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ABBREVIATIONS

PDF	Probability density function
UCM	Uncontrolled manifold theory

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ABSTRACT
The question of how to measure movement and then model has had a history of seeming simple at first and growing more complex. Richardson found that wind velocity challenged conventional Euclidean intuitions that supported linear modeling with whole-number dimensions. Biological movement coordination is no less complex than wind, and it has proven to warrant the same non-Euclidean treatment of its form. Dexterous movement requires a nonlinear spreading out of movement degrees of freedom across multiple spatiotemporal scales. This spreading out across scales exhibits what mathematicians call "divergence," meaning that the diffusion does not tidily wrap up at any single scale. Divergence entails an excess of dimensionality beyond the Euclidean imagination, from whole-number to fractional dimensionality and from single-fractional dimensionality to multiple. This perspective examines how fractional dimensions elucidate the intricate cascade of interactions underlying dexterous human movement. Traditional models, limited by linear and discrete representations, contrast with emerging multifractal approaches that capture continuous, context-sensitive variability across multiple interacting scales. Integrating multifractal analysis and cascade dynamics of human movement variability advances a new framework for understanding action across scales, highlighting its relevance for motor control and rehabilitation.

KEYWORDS: Cascade dynamics | Dexterity | Motor control | Movement variability | Multifractality | Scaling

Movement science has evolved significantly from early roots. Galileo and Newton provoked scientific revolution by applying Euclidean geometry to falling bodies. Fluid motion had more surprises in store. For instance, wind might not, strictly speaking, have velocity—not because it does not traverse finite space in finite time but rather because it diffuses with infinite capacity for heterogeneous texture (e.g., whirls within whirls) that spreads across infinite ranges of spatiotemporal scales [1]. Besides revolutionizing atmospheric science, this point would influence research on human movement. If puffs of wind defied simple horizontal and vertical description, how many more fluid violations of Euclidean intuitions would adaptive fluidity of biological movement commit? Early scientific efforts confined human action to discrete events, aiming Euclidean pretensions toward partitioning it into Boltzmannian state space. From gusts of wind to dexterous coordination of living perception-action cycles, movement science has begun to shed these Euclidean pretenses. The dawning realization is that modeling biological movement needs at least one fractional dimension and probably many more [2].

Today, movement scientists grapple with reconciling human movement's inherent flexibility and fluidity with the fixity of models. Models require fixity for generic utility, but traditional models have been slow to divest their fixity from linear and scale-bound assumptions of a vestigial Euclidean imagination. These Euclidean policies appear in the segmenting movement into isolated components (e.g., tiling time with "submovements" or Tinker-Toy approximations of degrees of freedom), invoking internal models [3], and attempting to reunite the components in an uncontrolled manifold (UCM) theory [4]. These models reduce continuous, adaptive processes into discrete snapshots assignable to specific degrees of freedom tuned to the precise timing of muscle activations or the specific kinematics of joint angles. Geometries that render the movement system into a serial chain neglect the fluid, cascading interactions across multiple scales wielding the organism's hierarchical form—from the microscopic fluctuations within muscle fibers to the macroscopic coordination of the entire body.

Multifractal modeling reflects a realization that, just as wind has no single velocity, actions are single-scale events. They allow the explicit acknowledgement of multiple-scale-invariant structures in movement coordination. These multifractal alternatives offer a modeling fixity through the explicit parametrization of diverse scaling relationships as diverse fractional dimensions capable of changing with time. Giving fractional-dimensional scaling relationships a mathematical voice through allows us to address generic and persistent structure in interactivity that Euclidean models cannot. Multiscale probability density function (PDF) analysis, for example, allows identifying specific scales at which these interactions become most pronounced [5-7]. So-called ``freezing'' and ``freeing'' of degrees of freedom are now understood to compose complex cascades, where micro-level fluctuations complement and constrain macro-level coordination [8]. Conventional Euclidean models and simple monofractal descriptions underspecify movement processes that thrive on interplay of multiple, interrelated scales. A gust of wind disturbed these models, and the human movement system bankrupts them entirely. Multifractal analysis and cascade dynamics offer a way to coordinate discrete measurements of the movement system as part of a continuous—if also heterogeneous—flow by recasting models of action in terms of fractional dimensions that evolve over time [2].

Multifractal models show early promises for fields such as robotics, rehabilitation, and sports science that share a growing need for non-invasvie interventions and devices that fit seamlessly with intrinsic human-movement variability [9]. For example, wearable exoskeletons and prosthetic devices stand to benefit from control algorithms parameterized according to multifractal cascades in human movement. Multifractal variability could inform rehabilitation protocols for adaptive treatments, helping patients regain functional movement by leveraging the body's wellspring of multifractal variability. Modeling the human movement is thus not simply an academic exercise: it frames how we understand and control movement for multiple settings and goals. The cost of Euclidean caricature outweighs the convenience of traditional linear modeling. Multifractal formalisms challenge us to interrogate our Euclidean notions of dimensionality, variability, and control. They invite integrative views of how we stitch together our behavior, for example, from coarsest scales of switching amongst multiple goals to all the finer-scaled sensory corrections and synergetic reconfiguring between.

In the struggle to reconcile discrete with continuous, simple with complex, the future of movement science lies in further integrating these advanced analytical frameworks with practical applications. The convergence of high-fidelity measurement technologies, sophisticated computational methods, and refined theoretical models promises to deepen our understanding of action at unprecedented levels of detail, e.g., combining multifractal analyses with machine learning to predict and modulate movement patterns [10]. The shift in focus towards multifractal dynamics and the cascade of interactions that underpin every movement offers a more complete and more productive picture of what it means to act.

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